# CS3933 Winter 2007 – Group Assignment 5
# Due Wednesday April 4, 2007, at 5pm.

1. We are planning to walk through a park whose trails are laid out as a grid of $n$ North-South trails and $m$ East-West trails, all intersecting. Each intersection $(i, j)$ has a corresponding altitude $a_{(i,j)}$. We need to travel from the Northwestern corner $(1, 1)$ to the Southeastern corner $(n, m)$, and will only travel East and South. However, there are many paths through these trails that preserve this requirement.

   We want to find a path through this park, traveling only East and South, that will minimise the total altitude climbed up (ignoring any climbing down). If we move from $(i, j)$ to its Eastern neighbour $(i, j + 1)$, we would add the difference in altitude $a_{(i,j+1)} - a_{(i,j)}$ if and only if this value is positive. Similarly, if we move from $(i, j)$ to its Southern neighbour $(i + 1, j)$, we would add the difference in altitude $a_{(i+1,j)} - a_{(i,j)}$ if and only if it is positive.

   According to this measurement scheme, if we have a path of altitudes $6, 3, 10, 7, 4, 8$, it would have a total climbed altitude of $(10 - 3) + (8 - 4) = 11$.

   Design a dynamic programming algorithm that will, given $n$, $m$, and the $n \cdot m$ altitude values $a_{(i,j)}$, find the path through the grid that minimises the total altitude climbed.

   Remember to find both the minimum altitude climbed and the path that produces it.

   Analyse the running time of your algorithm.

2. When finding a set of people to serve as representatives on a committee, it can be very important to ensure that no two representatives are from the same group. A person, of course, can be part of many different groups.

   We define the Committee Formation Problem as:
   Given: a set of people $P$, a set of subsets $S = \{S_1, \ldots, S_n\}$ where each $S_i \subseteq P$, and a positive integer $k$.
   Question: Is there a subset $P' \subseteq P$ with $|P'| \geq k$, where for each $S_i \in S$, at most one element of $P'$ is in $S_i$? (more precisely, we want $|S_i \cap P'| \leq 1$, $\forall S_i \in S$).

   Show that Committee Formation is equivalent to Independent Set, by:

   (a) Proving that Committee Formation $\leq_p$ Independent Set.

   (b) Proving that Independent Set $\leq_p$ Committee Formation.

   Note: Since it is very important that you understand how to construct the correct direction of a reduction, some marks will be deducted if you confuse (a) and (b). Clarify the direction with the professor if you are unsure.